Integrated Semantics for OWL Ontologies

Dorel Lucanu

Al. I. Cuza Univ., Iași, RO

September 18, 2007, NUS
Example: OWL ontology $O_1$

Class(a:Adult partial)
Class(a:American complete
    restriction(a:livesIn value (a:US)))
Class(a:Country partial)
Class(a:Exception partial)
Class(a:CarOwner complete
    intersectionOf(complementOf(a:Exception) a:American a:Adult))
ObjectProperty(a:livesIn Functional range(a:Country))
Individual(a:John type(a:Adult) value(a:livesIn a:US))
Individual(a:US type(a:Country))

Infered type (Protege + Racer) CarOwner = ∅
Example: $O_1$ as a LP with NAF semantics

country(us).
livesIn(john, us).
adult(john).
livesIn(nick, us).
exception(peter). % just for defining the predicate exception
american(X) :- livesIn(X, us).
carOwner(X) :- american(X), adult(X), \+ exception(X).

Ciao execution:

?- carOwner(X).
X = john ?
yes
Example: OWL ontology $O_2$

ObjectProperty(a:knownBy domain(a:foo) range(a:foo))

Class(a:famous complete restriction(a:knownBy someValuesFrom(complementOf(a:famous))))

Class(a:foo partial a:famous)

Individual(a:boby type(a:foo) value(a:knownBy a:john))
Individual(a:john type(a:foo) value(a:knownBy a:boby))
Individual(a:peter type(a:foo) value(a:knownBy a:john) value(a:knownBy a:boby))

Inferred type (Protege + Racer) famous = Ǿ
Example: $O_2$ as a LP (NAF semantics)

```
foo(john).  
foo(boby).  
fooPerson(peter).  
knownBy(john, boby).  
knownBy(boby, john).  
knownBy(peter, john).  
knownBy(peter, boby).  
famous(X) :- knownBy(X, Y), \+famous(Y).  
```

Ciao execution:

```
?- famous(X).  
% Malloc: No such file or directory
ERROR: Memory allocated out of addressable bounds!
ERROR: segmentation violation
```
Example: $O_2$ as a LP (stable semantics)

AnsProlog:

<table>
<thead>
<tr>
<th>Answer: 1</th>
<th>Answer: 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>famous(peter)</td>
<td>famous(peter)</td>
</tr>
<tr>
<td>famous(boby)</td>
<td>famous(john)</td>
</tr>
<tr>
<td>knownBy(peter,boby)</td>
<td>knownBy(peter,boby)</td>
</tr>
<tr>
<td>knownBy(peter,john)</td>
<td>knownBy(peter,john)</td>
</tr>
<tr>
<td>knownBy(boby,john)</td>
<td>knownBy(boby,john)</td>
</tr>
<tr>
<td>knownBy(john,boby)</td>
<td>knownBy(john,boby)</td>
</tr>
<tr>
<td>foo(peter)</td>
<td>foo(peter)</td>
</tr>
<tr>
<td>foo(boby)</td>
<td>foo(boby)</td>
</tr>
<tr>
<td>foo(john)</td>
<td>foo(john)</td>
</tr>
</tbody>
</table>

$\vdash$ famous(peter)
Example: \( O_2 \) as a LP (stable semantics)

AnsProlog:

Answer: 1
- \text{famous}(peter)
- \text{famous}(boby)
- \text{knownBy}(peter, boby)
- \text{knownBy}(peter, john)
- \text{knownBy}(boby, john)
- \text{knownBy}(john, boby)
- \text{foo}(peter)
- \text{foo}(boby)
- \text{foo}(john)

\[ \vdash \text{famous}(peter) \]

Answer: 2
- \text{famous}(peter)
- \text{famous}(john)
- \text{knownBy}(peter, boby)
- \text{knownBy}(peter, john)
- \text{knownBy}(boby, john)
- \text{knownBy}(john, boby)
- \text{foo}(peter)
- \text{foo}(boby)
- \text{foo}(john)
Example: remarks

- OWL semantics interpret ontologies under Open World Assumption (OWA): $O \models \neg \varphi$ iff all $O$-models interpret $\varphi$ as being false. $O \models \text{CarOwner}(\text{John})$ iff John \notin \text{Exception} in all $O$-models.

- LP semantics interpret ontologies under Closed World Assumption (CWA): $O \models \neg \varphi$ iff NOT $O \vdash \varphi$.

- Various LP semantics interpret some OWL ontologies (e.g., $O_1$) in the same way, others (e.g., $O_2$) in a different way. Why?

- $O_1$: NAF semantics = stable semantics

- $O_2$: NAF semantics \neq stable semantics (\text{knownBy} is cyclic)

- When we believe an LP-based agent? (Is John a CarOwner?)

- How closing semantics can be expressed in OWL terms?
We are able to answer the previous questions if we know

- what is a logic
- what is the logic corresponding to the interpretation under CWA
- what means a translation between two logics
  OWA - OWA
  OWA - CWA
- how to use modal operators to close semantics
What is a Logic?

A possible answer is given by institutions (Goguen & Burstall, 1984)

- a collection of signatures: vocabularies
- a collection of models: structures interpreting the symbols (names) from a signature
- a collection of sentences: formulas built with symbols from a signature intended to express specific properties
- a satisfaction relation: says when a given sentence holds in a given model (both corresponding to the same signature)
What is a Logic?

Institution of Description Logic (DL)

- a signature consists of
  class names: $CN = \{ \text{Country, American}, \ldots \}$
  property names: $PN = \{ \text{livesIn} \}$
  individual names: $IN = \{ \text{John} \}$

- morphism of signatures $\phi : \Sigma \rightarrow \Sigma'$
  class names $\rightarrow$ class names (e.g., $\phi(\text{American}) = \text{AmericanCitizen}$)
  property names $\rightarrow$ property names (e.g., $\phi(\text{livesIn}) = \text{hasResidence}$)
  individual names $\rightarrow$ individual names (e.g., $\phi(\text{John}) = \text{JohnWalker}$)
What is a Logic?

DL Institution

- **a model** consists of
  - a carrier set $\Delta$, e.g., Reals
  - a class name is interpreted as a subset of $\Delta$
    - e.g. $\llbracket Country \rrbracket = \text{Integers}$, $\llbracket American \rrbracket = \text{Irrational numbers}$
  - a property name is interpreted as a set of pairs, e.g.,
    - $\llbracket livesIn \rrbracket = \{(x, [x]) \mid x \notin \text{Integers}\}$
  - an individual name is interpreted as an element of $\Delta$
    - e.g. $\llbracket John \rrbracket = 3.1415\ldots$

- **model morphisms** $h : M \rightarrow M'$, e.g., $h : \text{Reals} \rightarrow \text{Complex}$
  (perhaps is difficult to find a morphism between Reals and People $\cup$ Countries)

- if $\phi : \Sigma \rightarrow \Sigma'$ and $M'$ is a $\Sigma'$-model, then we can define the $\Sigma$-model $M'|_{\phi}$, which is $M'$ viewed as a $\Sigma$-model

Remark: It is more realistic for OWL to restrict the interpretation to
Herbrand models; OWA is preserved (Fitting, 1996).
DL Institution

- **sentences** express relations between class/properties expressions, e.g., $\forall \text{livesIn.\{US\}} \sqsubseteq \text{American}$

- **signature morphisms** are extended to **sentence translations**, e.g., $\phi(\forall \text{livesIn.\{US\}} \sqsubseteq \text{American}) = \forall \text{hasResidence.\{USCountry\}} \sqsubseteq \text{AmericanCitizen}$

- **satisfaction relation** $M \models_\Sigma \forall \text{livesIn.\{US\}} \sqsubseteq \text{American}$
  it must satisfy the satisfaction condition:
  $M' \models_{\Sigma'} \forall \text{hasResidence.\{USCountry\}} \sqsubseteq \text{AmericanCitizen}$ if
  $M'|_\phi \models_\Sigma \forall \text{livesIn.\{US\}} \sqsubseteq \text{American}$
  the truth is invariant under change of notation
What is a Logic Morphism?

- **institution morphisms** - how a richer logic is built over a poorer one, e.g., \((\Phi, \beta, \alpha) : \text{FOL} \to \text{HL}\)

- **institution comorphisms** - how a logic is encoded in another logic \((\Phi, \beta, \alpha) : \text{DL} \to \text{FOL}\)
  - \(\Phi\) encodes DL signatures into FOL signatures
  - \(\beta\) interpret FOL \(\Phi(\Sigma)\)-models as DL \(\Sigma\)-models
  - \(\alpha\) translates DL \(\Sigma\)-sentences into FOL \(\Phi(\Sigma)\)-sentences

- a satisfaction condition must be satisfied: a FOL \(M'\) model satisfies \(\alpha(\varphi)\) iff the DL model \(\beta(M')\) satisfies \(\varphi\)
the signatures are logic programs (GHL specifications), e.g., $P_1$:

$$\text{american}(X) \leftarrow \text{livesIn}(X, \text{us}).$$

$$\text{carOwner}(X) \leftarrow \text{american}(X), \text{adult}(X), \text{not} \ Exception(X).$$

$$\text{country}(\text{us}).$$

$$\text{livesIn}(\text{john, us}).$$

$$\text{adult}(\text{john}).$$
An LP paradigm associates a set of models $SEM(P)$ to a logic program $P$
E.g., stable semantics associates to $P_1$ the uniques stable model $M_1$:

Answer: 1
carOwner(john)
adult(john)
american(john)
livesIn(john,us)
country(us)

$P_1 \models^{STAB} \text{carOwner}(\text{john})$ and hence any model in $\widehat{\text{GLP}}^{STAB}$ must satisfy
\text{carOwner}(\text{john})$. 
Let $P_2$ denote the logic program $P_1 \cup \{\text{exception(john)}\}$. $P_2$ has also a unique stable model, denoted by $M_2$:

Answer: 1
exception(john)
adult(john)
american(john)
livesIn(john,us)
country(us)

$M_2$ is not a model of $P_1$ in the sense of the stable semantics and therefore the inclusion $P_1 \subset P_2$ is NOT a signature morphism in $\widehat{\text{GLP}^{\text{STAB}}}$.
If we consider the program \( P_3 = P_1 \cup \{american(peter)\} \), then its stable model is \( M_3 = M_1 \cup \{american(peter)\} \), which is a consistent extension of \( M_1 \) and therefore can be seen as a model for \( P_1 \).

The functor \( \text{Mod}(\hat{\text{GLP}}^\bullet) \) associates to a program those extended Herbrand models which are consistent extensions of models given by \( \text{SEM}^\bullet \) (the Hebrand universe is extended with an infinite set of constants).

A FOL morphism \( \varphi : P \to P' \) is a signature morphism in \( \hat{\text{GLP}}^\bullet \) if and only if for each \( P' \)-model \( M' \in \text{Mod}^\bullet(P') \), its reduct \( M'|_{\varphi} \) is a \( P \)-model.

In this the satisfaction condition is automatically satisfied.
Let $KB_1$ be the following DL knowledge base (DL semantics of $O_1$):

\[
\begin{align*}
Country & \sqsubseteq \top \\
Adult & \sqsubseteq \top \\
Exception & \sqsubseteq \top \\
American & \equiv \exists \text{\textit{livesIn}}.\{\text{US}\} \\
Adult \sqcap American \sqcap (\neg Exception) & \sqsubseteq \text{carOwner}
\end{align*}
\]

\[
\begin{align*}
John & \in \text{adult} \\
US & \in \text{country} \\
\langle John, US \rangle & \in \text{livesIn}
\end{align*}
\]

$KB_1$ can be translated into logic program $P_1$.

We have $P_1 \models^{STAB} \text{carOwner}(john)$ but $KB_1 \not\models^{DL} John \in \text{CarOwner}$
Translation of DL in GLP

\( KB_2 = KB_1 \cup \{ john \in \text{exception} \} \)

\( P_2 \) is a translation of the \( KB_2 \) as a logic program.

\( KB_1 \subset KB_2 \) is a signature morphism in \( \widehat{DL}^{th} \) but the inclusion \( P_1 \subset P_2 \) is not a morphism in \( \widehat{GLP}^{STAB} \).

Therefore we consider a subinstitution \( \widehat{DL}^{th,STAB} \), where only translatable morphisms are kept.

We can define now a comorphism \( (\Phi, \beta, \alpha) : \widehat{DL}^{th,STAB} \rightarrow \widehat{GLP}^{STAB} \).
Integrated semantics

For the OWL ontology $O_1$ we have two semantics:

- DL semantics ($KB_1$)
- GLP$^{STAB}$ semantics ($P_1$)

Consider again $(\Phi, \beta, \alpha) : \widehat{DL}^{th,STAB} \rightarrow \widehat{GLP}^{STAB}$.

Since $\text{Mod}(\widehat{GLP}^{STAB})(\Phi(KB_1) = P_1) \subseteq \text{Mod}(\widehat{DL}^{th,STAB})(KB_1)$, the above comorphism close the semantics of DL according to stable semantics of LP.

Can the two semantics live happily together into an integrated semantics for OWL?
Integrated semantics

Consider the diagram:

\[
\begin{array}{ccc}
\text{DL} & \text{DL} & \text{STAB} \\
\text{th} & \text{th} & \text{STAB} \\
\text{GLP} & \text{STAB} \\
\end{array}
\]

Let \( \text{OWL}^D \) be the semi-institution consisting of a pair \((\text{Sign}(\text{OWL}^D), \text{sen}(\text{OWL}^D))\), where \(\text{Sign}(\text{OWL}^D)\) is subcategory of OWL ontologies which can be translated in signatures in \(\widehat{I}\), for all \(\widehat{I} \in |D|\).

An interpretation \(i(\widehat{I}) : \text{OWL}^D \rightarrow \widehat{I}\) is a pair \((\Phi, \alpha)\) defined in the same way as for institution comorphisms.

An \(O\)-model in \(\text{Mod}(\text{OWL}^D)(O)\) is a pair \((i, M)\), where \(i = (\Phi, \alpha) : \text{OWL}^D \rightarrow \widehat{I}\) is a interpretation and \(M\) is a \(\Phi(O)\)-model in \(\widehat{I}\).

We have a model \(O\)-morphism \(h : (i, M) \rightarrow (i', M')\) if and only if \(i = (\Phi, \alpha) : \text{OWL}^D \rightarrow \widehat{I}\), \(i' = (\Phi', \alpha') : \text{OWL}^D \rightarrow \widehat{I}'\), and there is a comorphism \((\Phi, \beta, \alpha) : \widehat{I} \rightarrow \widehat{I}'\) such that \(\beta(M') = M\).
A DL-based agent may interoperable with a GLP\textsuperscript{*}-based agent only if their models are related by the functor $\beta$.

Can $\beta$ be syntactically represented? If yes, then it can be handled in SW.

The answer $\text{carOwner}(\text{john})$ given by $P_1$ must be read as “assuming that John is not an Exception, knowing that John is American, and knowing that John is an Adult, then we know that John is a CarOwner.

This can be expressed by the following MKNF rule:

\[
K \text{ carOwner}(X) \leftarrow K \text{ american}(X), K \text{ adult}(X), \text{not Exception}(X) \tag{1}
\]

where $K A$ is read as “$A$ is known to hold” and $\text{not} A$ as “is it possible for $A$ not to hold”.
So, endowing $O_1$ with a $\text{GLP}^{\text{STAB}}$ semantics, we get back $O_1$ together with the constraint given by rule (1).

**OWL DL ontologies together MKNF rule = Hybrid MKNF knowledge bases** (Motik & Rosati, 2006)

$$\beta(\text{Mod}(\hat{\text{GLP}^{\text{STAB}}}(P_1))) = \text{Mod}(\hat{\text{HMKNFKB}})(O_1, \{(1)\})).$$

I.e., $O_1$ together with LP stable models is semantically equivalent with the hybrid MKNF kb consisting of $O_1$ and the rule (1).
Conclusion

- OWL ontologies can be interpreted using various logics
- ontology morphisms must also be taken into account
- different interpretations are related via comorphisms and together supplies an integrated semantics for OWL ontologies
- the DL reflection of an OWL DL ontology with LP stable semantics is equivalent with an hybrid MKNF knowledge base
- this result can be extended to other logics interpreting OWL under CWA (not only LP)
- integrated semantics can be enriched using limits/colimits
Thanks!